1 Electromagnetic Waves

Maxwell's equations in differential form are given by the following:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{4}$$

Write down Maxwell's equations with no sources present

Using these equations, derive the following properties of electromagnetic radiation

- The electric field obeys the wave equation
- The magnetic field obeys the wave equation
- The speed of propagation for each of these fields is $c = 1/\sqrt{\mu_0 \epsilon_0}$

Hint: The following vector identity may help.

$$\nabla \times (\nabla \times \mathbf{V}) = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V} \tag{5}$$

The wave equation is a linear equation, and so it obeys superposition. In particular, we can decompose the solution to the electric and magnetic fields in terms of a sum (or integral) over all wavevectors. In the following, we consider a specific wavevector $\mathbf{k}$. We can write down the solution for the electric and magnetic fields for this mode as the following.

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)} \tag{6}$$

$$\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t + \psi)} \tag{7}$$

$\mathbf{E}_0$ and $\mathbf{B}_0$ are real amplitudes. To get the physical fields, the real components of these modes must be taken.

Derive the following properties of this mode

- This mode is a solution to the wave equations you derived above
- The electric and magnetic fields have the same frequency of oscillation (how does this relate to $\mathbf{k}$?)
- The electric and magnetic fields oscillate in phase
- $\mathbf{E}$ and $\mathbf{B}$ and transverse to $\mathbf{k}$
- The relationship $c\mathbf{B} = \hat{\mathbf{k}} \times \mathbf{E}$
- The electric field and magnetic field have amplitudes that are related (what is this relation?)
- The electric field is transverse to the magnetic field

Use Poynting's theorem to describe the average intensity of this wave

Use Poynting's theorem to describe the average momentum carried by the wave

This wave is reflected off of a mirror.

Calculate the pressure exerted on the mirror by the wave